Solution to Worksheet - 5

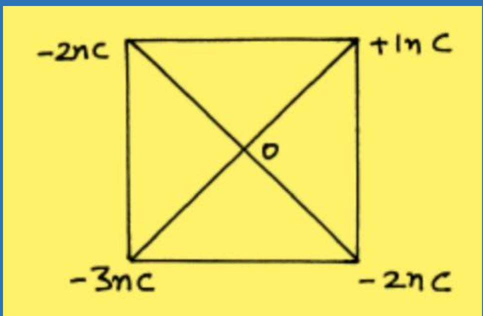
A.1 $V_A + V_B = 0$

$$\frac{1}{4\pi\epsilon_0}\left[\frac{q_A}{r_A} + \frac{q_B}{r_B}\right] = 0$$

$$\left[\frac{4}{x} - \frac{2}{(1-x)}\right] = 0$$

$$4 = 6x \implies x = 0\cdot 67\ \text{m}$$

A.2 $V_0 = V_A + V_B + V_c + V_D$



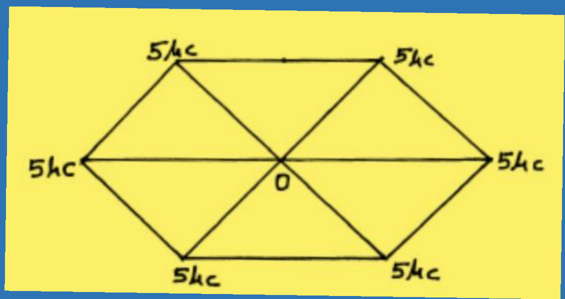
$$V_0 = \frac{1}{4\pi\epsilon_0} \times 10^{-9}[q_A + q_B + q_C + q_D]$$

$$V_0 = 9 \times 10^9 \times 10^{-9}[-2+1-2-3]$$

$$V_0 = -54\ \text{V}$$

A.3 $V_0 = 6 \times V_A$

OD = OC = OA = OB = OF = OE = 10 cm



$$V_0 = 6 \times \frac{1}{4\pi\epsilon_0} \times \frac{q_A}{r_A}$$

$$V_0 = \frac{6 \times 9 \times 10^9 \times 5 \times 10^{-6}}{\frac{10}{100}} = 2\cdot 7 \times 10^6\ \text{V}$$

A.4 $\vec{E}$ will be O at the centre of the cube·

Potential at O due to + q charge:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{1}{4\pi\epsilon_0} \times \frac{q}{\frac{\sqrt{3}b}{2}}$$

∴ Net potential at O = 8V

$$= \frac{8 \times 2q}{4\pi\epsilon_0\sqrt{3}b}$$

$$V_{net} = \frac{4q}{\sqrt{3}\pi\epsilon_0 b}$$

A.5 Radius of smaller drop r = 1mm

R – radius of bigger drop

$$27 \times \frac{4\pi}{3} r^3 = \frac{4\pi}{3} R^3$$

$$R = 3r = 3 \times 1 = 3\ \text{mm}$$

$$R = 3 \times 10^{-3}\text{m}$$

Charger on bigger drop q = 27×10^{-12} C

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$$

$$= \frac{9 \times 10^9 \times 27 \times 10^{-12}}{3 \times 10^{-3}}\ \text{Volt}$$

$$= \mathbf{81\ V}$$

A.6 Work done in moving charge B 1 cm towards charge A =

$$W_{BA} = \frac{1}{4\pi\epsilon_0} qq_0 \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$W_{BA} = 9 \times 10^9 \times 5 \times 10^{-8} \times 3 \times 10^{-9}\left[\frac{1}{.05} - \frac{1}{.06}\right]$$

$$\mathbf{W_{BA} = 4\cdot 5 \times 10^{-6}\ J}$$

A.7 **a)** $E = 2000\frac{N}{C},\ d = 0\cdot 2\ \text{m}$

$$E = \frac{V}{d}\ \ or\ \ V = E \times d$$

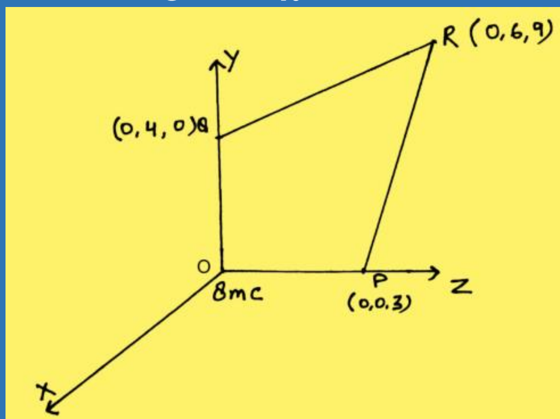
$$V = 2000 \times 0{\cdot}2 = 400\ \text{V}$$

b) $V = \frac{W}{q}\ \ or\ \ W = V \times q$

$$W = 400 \times 1\cdot 6 \times 10^{-19}\ \text{J}$$

$$W = 6\cdot 4 \times 10^{-17}\ \text{J}$$

A.8 Charge at O, $q_0 = -2 \times 10^{-19}$ C



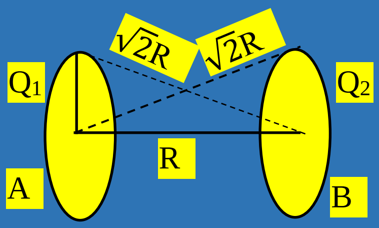
Work done in moving q from P to Q

$$W_{PQ} = \frac{1}{4\pi\epsilon_0} qq_0 \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$W_{PQ} = 9 \times 10^9 \times 8 \times 10^{-3} \times -2 \times 10^{-9}\left[\frac{1}{.04} - \frac{1}{.03}\right]$$

$$W_{PQ} = -1\cdot 2\ \text{J}$$

A.9



Potential at A, $V_A = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{R} + \frac{1}{4\pi\epsilon_0}\frac{Q_2}{\sqrt{2}R}$

Potential at A, $V_B = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{R} + \frac{1}{4\pi\epsilon_0}\frac{Q_1}{\sqrt{2}R}$

Work done in taking charge q from center of ring A to ring B, $W = q\,(V_B - V_A)$

$$W = q\left[\frac{1}{4\pi\epsilon_0 R}\left\{(Q_2 - Q_1) + \frac{Q_1 - Q_2}{\sqrt{2}}\right\}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{R}(Q_2 - Q_1) + \left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q(Q_2 - Q_1)(\sqrt{2} - 1)}{\sqrt{2}\ R}$$

A.10 $\overline{E} = (20\,\hat{i} + 30\,\hat{j})$ N/C

$V(0,0) = 0$

$V(2,2) = ?$

$V = -\vec{E}.\vec{d}$

$V = -(20\,\hat{i} + 30\,\hat{j}).(2\,\hat{i} + 2\,\hat{j})$

$V = -100$ V

A.11 $|\vec{E}| = \frac{V}{d}$

$2 \times 10^3 = \frac{V}{4}$

KE = Workdone = qV

$= 3 \times 10^{-6} \times 8 \times 10^{+3}$

$= 24 \times 10^{-3}$ J

$V(4m) - V(0) = -8 \times 10^3$ V

$\because \left(E = -\frac{dv}{dr}\right)$

A.12 $E = -\frac{dV}{dx}$

$V(x) = \frac{20}{(x^2 - 4)}$ V

$E = -\frac{d}{dx}\left(\frac{20}{x^2 - 4}\right)$

$E = -20\,(-1)\,(x^2 - 4)^{-2}\,(2x)$

$E = \frac{40x}{(x^2 - 4)^2}$

$E = \frac{40 \times 4}{(12)^2} \because (x = 4\mu\,m)$

$E = \frac{10}{9}\frac{V}{\mu m} = 1.1\frac{V}{\mu m}$

A.13 $+\sigma = \frac{q_A}{4\pi a^2}$

$q_A = \sigma\,4\pi a^2$

$q_B = -\sigma\,4\pi b^2$

$q_C = -\sigma\,4\pi c^2$

Potential of shell A,

V_A = pot. due to q_A + pot. due to q_B + pot. due to q_C

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{\sigma\,4\,\pi a^2}{a} + \frac{(-\sigma\,4\,\pi b^2)}{b} + \frac{\sigma\,4\pi\,c^2}{C}\right]$$

$$= \frac{\sigma}{\epsilon_0}(a - b + c)$$

Potential of shell B,

V_B = pot. due to $(q_A + q_B)$ + pot. due to q_C

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{\sigma\,4\,\pi a^2 - \sigma\,4\,\pi b^2}{b} + \frac{\sigma\,4\pi\,c^2}{C}\right]$$

$$= \frac{\sigma}{\epsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$

Potential of shell C

$$V_C = \frac{1}{4\pi\epsilon_0}\frac{q_A + q_B + q_C}{C}$$

$$= \frac{\sigma}{\epsilon_0}\left(\frac{a^2 - b^2 + c^2}{C}\right)$$

A.15 U of Δ ABC $= U_{AB} + U_{BC} + U_{AC}$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{3 \times 1}{1} + \frac{3 \times 2}{1} + \frac{1 \times 2}{1}\right]$$

$U = 99 \times 10^9$ J

U of Δ DEF $= U_{DE} + U_{EF} + U_{DF}$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{1 \times 3}{0{\cdot}5} + \frac{3 \times 2}{0{\cdot}5} + \frac{1 \times 2}{0{\cdot}5}\right]$$

$U = 1\cdot98 \times 10^9$ J

$W = U_{ABC} - U_{DEF}$

$W = 99 \times 10^9 - 1\cdot98 \times 10^9$

$W = 10^{11}$ J