Solution to Worksheet – 1

A.1 $q_1 = -3 \times 10^{-12}$ C

$q_2 = +8 \times 10^{-12}$ C

$q_3 = +4 \times 10^{-12}$ C

a) Since the balls are brought in contact, sharing of charges will take place

$$q_{net} = \frac{q_1 + q_2 + q_3}{3}$$

$$q_{net} = 3 \times 10^{-12} \text{ C}$$

b) $q = \pm ne$ or $n = \frac{q_{net}}{e}$

$\therefore n = 1 \cdot 87 \times 10^7$

A.2 $q_1 = 2 \cdot 5\,\mu c, q_2 = -1 \cdot 6\,\mu c$

F = 0·4 N, r = ?

a) $F = \frac{1}{4\pi \epsilon_0} \cdot \frac{|q_1|\,|q_2|}{r^2}$

$$r^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{|q_1|\,|q_2|}{F}$$

Plug in the values and get the result

$r = 0 \cdot 3$ m

b) $r' = r + \frac{20}{100} \times r = 1.2r$

$r' = 1 \cdot 2r$

% decrease in force =

$$\frac{\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} - \frac{1}{4\pi\epsilon_0}\frac{q^2}{(1.2r)^2}}{\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}} \times 100$$

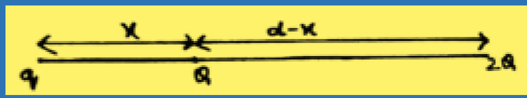
$= \frac{1{\cdot}44 - 1}{1{\cdot}44} \times 100 = 30.5\%$

A.3 $F = \frac{1}{4\pi\epsilon_0} \frac{|q_1|\,|q_2|}{F}$ - (I)

$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{|q_1|\,|q_2|}{(d')^2}$ - (II)

$\frac{eq.II}{eq.I} \Rightarrow d' = \frac{d}{3}$ m

A.4 For the net force to be zero, Q charge should be – ve and be placed in b/w the other 2 charges·



$$\left|\overrightarrow{F_{CA}}\right| = \left|\overrightarrow{F_{CB}}\right|$$

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{|q|\,|Q|}{x^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{|2q|\,|Q|}{(d-x)^2}$$

$$\frac{1}{x^2} = \frac{2}{(d-x)^2}$$

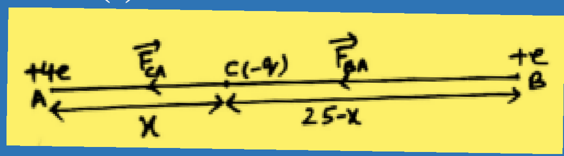
$$\frac{1}{x} = \frac{\sqrt{2}}{(d-x)}$$

$$x = \frac{d}{\sqrt{2}+1}$$

$$x = \frac{(\sqrt{2}-1)d}{(\sqrt{2}+1)(\sqrt{2}-1)}$$

$$\therefore x = \left(\sqrt{2}-1\right) d$$

A.5 (a)



The charge – q should be placed at C.

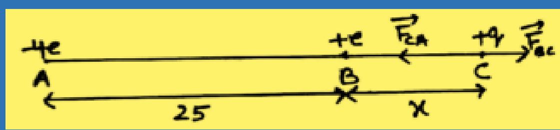
For the system to be in equilibrium,

$$\left|\overrightarrow{F_{CA}}\right| = \left|\overrightarrow{F_{CB}}\right|$$

$$\frac{1}{4\pi\epsilon_0} \frac{|q_A|\,|q_C|}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{|q_A|\,|q_C|}{(25-x)^2}$$

$\frac{4e}{x^2} = \frac{e}{(25-x)^2} \Rightarrow x = 16.67$cm

(b)

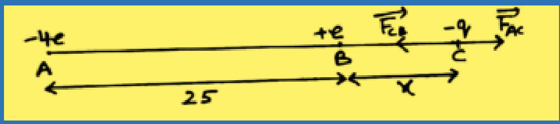


$$\left|\overrightarrow{F_{CA}}\right| = \left|\overrightarrow{F_{BC}}\right|$$

$$\frac{1}{4\pi\epsilon_0} \frac{|q_A|\,|q_C|}{(25+x)^2} = \frac{1}{4\pi\epsilon_0} \frac{|q_B|\,|q_C|}{x^2}$$

$$x = 25 \text{ cm}$$

(c)

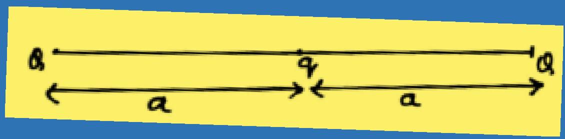


$$|\vec{F_{AC}}| = |\vec{F_{CB}}|$$

$$\frac{1}{4\pi\epsilon_0}\frac{|q_A|\,|q_C|}{(25+x)^2} = \frac{1}{4\pi\epsilon_0}\frac{|q_B|\,|q_C|}{x^2}$$

$$x = 25 \text{ cm}$$

A.6



Net force at A = 0

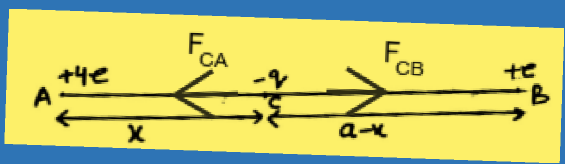
$$F = F_{CA} + F_{AB}$$

$$0 = \frac{1}{4\pi\epsilon_0}\frac{q\,Q}{Q^2} + \frac{1}{4\pi\epsilon_0}\frac{Q^2}{(2a)^2}$$

$$0 = Q\,(4q + Q)$$

$$4q + Q = 0 \Longrightarrow q = \frac{-Q}{4}$$

A.7 The sign of q will be – ve to keep the entire system in equilibrium.



$$|\vec{F_{CA}}| = |\vec{F_{CB}}|$$

$$\frac{1}{4\pi\epsilon_0}\frac{|4e|\,|q|}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{|e|\,|q|}{(a-x)^2}$$

$$2a - 2x = x \Rightarrow x = \frac{2a}{3}$$

A.8 Q is divided into 2 parts —q &Q

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q|\,|Q-q|}{r^2}$$

For F to be max $\frac{dF}{dq} = 0$

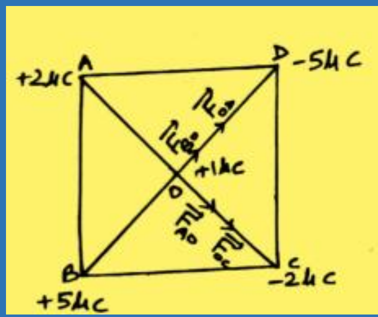
$$Q - 2q = 0 \Longrightarrow q = \frac{Q}{2}$$

$$\frac{d^2F}{dq^2} = 0 - 2 < 0 \qquad \forall q$$

$\Longrightarrow$ at $q = \frac{Q}{2}$, F is max·

Hence, the charges are $\frac{Q}{2}, \frac{Q}{2}$·

A.9



$$F_1 = F_{AO} + F_{OC} = 2\,F_{AO}$$

$$F_1 = 2\left(\frac{1}{4\pi\epsilon_0} \times \frac{5\times10^{-6}\times1\times10^{-1}}{\left(\frac{0\cdot1\sqrt{2}}{2}\right)^2}\right)$$

$$\mathbf{F_1 = 7\cdot2\ N}$$

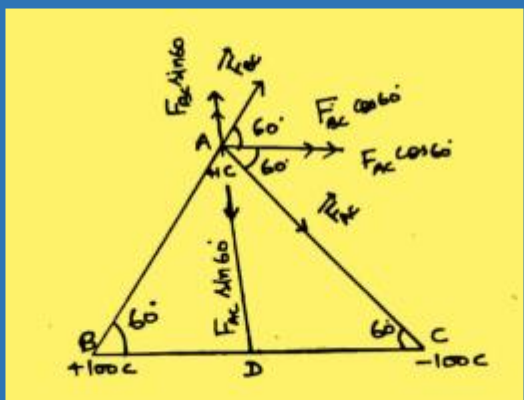
$$F_2 = F_{BO} + F_{OD} = 2\,F_{BO}$$

$$F_2 = 2\left(\frac{1}{4\pi\epsilon_0} \times \frac{5\times10^{-6}\times1\times10^{-1}}{\left(\frac{0\cdot1\sqrt{2}}{2}\right)^2}\right)$$

$$\mathbf{F_2 = 18\ N}$$

$$\therefore F_{net} = \sqrt{F_1{}^2 + F_2{}^2} = \mathbf{193\cdot38\ N}$$

A.10



$$F_{AC} = \frac{1}{4\pi\epsilon_0} \times \frac{1\times100}{100}$$

$$\mathbf{F_{AC} = 9 \times 10^9\ N}$$

$$F_{BA} = \frac{1}{4\pi\epsilon_0} \times \frac{100 \times 1}{100}$$

$$\mathbf{F_{BA} = 9 \times 10^9\ N}$$

$$|\vec{F}_{BA}| = |\vec{F}_{AC}|$$

$$F_{DA} = \frac{1}{4\pi\epsilon_0} \times \frac{75 \times 1}{(5\sqrt{3})^2}$$

$$\mathbf{F_{DA} = 9 \times 10^9\ N}$$

$$F_x = F_{AC}\cos 60° + F_{BA}\sin 60°$$

$$F_x = 2 \times 9 \times 10^9 \times \frac{1}{2}$$

$$\mathbf{F_x = 9 \times 10^9\ N}$$

$$F_y = F_{DA} + F_{BA}\sin 60°$$

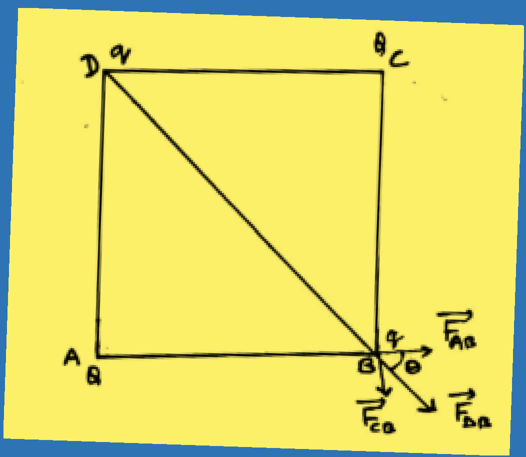
$$- F_{AC}\sin 60°$$

$$\mathbf{F_y = 9 \times 10^9\ N}$$

$$|F_{net}| = \sqrt{F_x^{\ 2} + F_y^{\ 2}}$$

$$\mathbf{F_{net} = 9\sqrt{2} \times 10^9\ N}$$

A.11



Resultant of F_{AB} & F_{CB}

$$F_1 = \sqrt{(F_{AB})^2 + (F_{CB})^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Qq}{a^2}\sqrt{2}$$

In ΔBPT $\qquad \tan\theta = \frac{F_{CB}}{F_{AB}} = 1$

$$\therefore\ \theta = 45$$

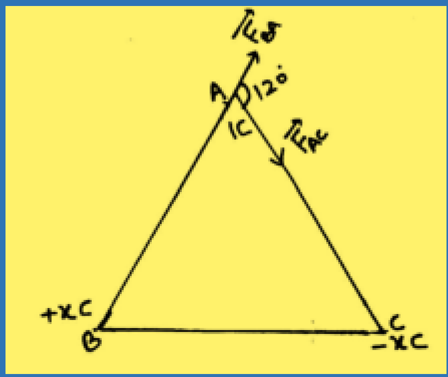
Now net force on B, $F_1 + F_{DB} = 0$

$$\frac{1}{4\pi\epsilon_0}\frac{Qq}{a^2}\sqrt{2} + \frac{1}{4\pi\epsilon_0}\frac{q^2}{(\sqrt{2}a)^2} = 0$$

$$\frac{Q\sqrt{2}}{a^2} + \frac{q}{2a^2} = 0$$

$$Q\sqrt{2} = \frac{-q}{2} \qquad \therefore q = -2\sqrt{2}\,Q$$

A.12 (i)



$$|\vec{F}_{AB}| = \frac{1}{4\pi\epsilon_0}\frac{1 \times x}{x^2}$$

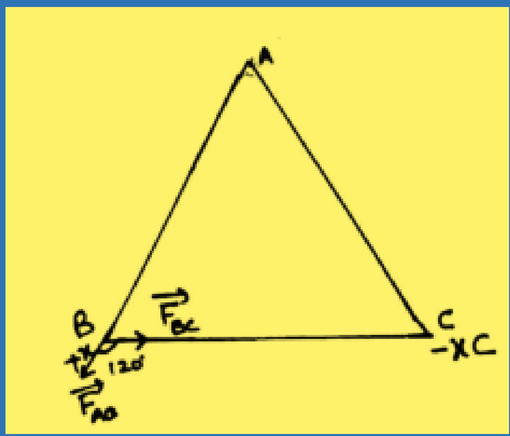
$$= \frac{1}{4\pi\epsilon_0\, x} = |\vec{F}_{AC}|$$

Net force on 1 C charge

$$F = \sqrt{F_{AB}^{\ 2} + F_{AC}^{\ 2} + 2F_{AB}F_{AC}\cos 120}$$

$$\Rightarrow F = \frac{1}{4\pi\epsilon_0\, x}$$

(ii)



$$F_{BC} = \frac{1}{4\pi\epsilon_0}\frac{x^2}{x^2} \qquad = \frac{1}{4\pi\epsilon_0}$$

Net force on $+\ x$ C at B

$$F = \sqrt{F_{AB}^{\ 2} + F_{BC}^{\ 2} + 2F_{AB}F_{AC}\cos 120}$$

$$= \sqrt{\left(\frac{1}{4\pi\epsilon_0 x}\right)^2 + \left(\frac{1}{4\pi\epsilon_0}\right)^2 + 2\left(\frac{1}{4\pi\epsilon_0 x}\right)\left(\frac{1}{4\pi\epsilon_0}\right)\left(-\frac{1}{2}\right)}$$

$$F = \left(\frac{1}{4\pi\epsilon_0}\right)\sqrt{x^{-2} + 1 - x^{-1}}$$

(iii) same as in (ii)

A.13 $F = mg$

$$F = 50 \times 10 = 500\ N$$

$$q_1 = q_2 = 1\cdot 0\ C, r = ?$$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1|\,|q_2|}{r^2}$$

$$500 = \frac{9 \times 10^9 \times 1 \times 1}{r^2}$$

$$r^2 = 18 \times 10^6$$

$$r = \pm\sqrt{18 \times 10^6}$$

$$r = 4\cdot 2 \times 10^3 m$$

A.14 Since the charges attract each other with F = 0·1 N, the charges with F = 0·1 N, the charges will be equal in magnitude but opposite in sign.

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q|\,|q|}{r^2}$$

$$q^2 = \frac{0\cdot 1 \times \left(\frac{1}{100}\right)^2}{9 \times 10^9}$$

$$q = \pm\sqrt{\frac{1}{9} \times 10^{-14}}$$

$$q = \pm 3\cdot 3 \times 10^{-8} C$$

Now, $q = \pm n e$

$$n = \frac{q}{e} = \frac{3\cdot 3 \times 10^{-8}}{1\cdot 6 \times 10^{-19}}$$

$$\therefore n = 2 \times 10^{11}$$

A.15 $q_A = q_B = 2 \times 10^{-8}\ C$

$$T = F$$

$$T = \frac{1}{4\pi\epsilon_0}\frac{|q_A|\,|q_B|}{r^2}$$

$$T = \frac{9 \times 10^9 \times (2 \times 10^{-8})^2}{1 \times 1}$$

$$T = 3\cdot 6 \times 10^{-6}\ N$$

A.16

$$T\cos\theta = mg - (1)\ \&$$

$$T\sin\theta = F$$

$$T\sin\theta = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(2L\sin\theta)^2} - (2)$$

1/2 gives

$$\tan\theta = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(2L\sin\theta)^2}/mg$$

$$q^2 = 4\,mg\,L^2\,\sin\theta\tan\theta\,(4\pi\varepsilon_0)$$