Trigo at a glance

Formulae related to angle

$1° = 60'$ and $1' = 60''$

1. Relation between degree and radian:

2 right angle = $180° = \pi$ radians

2. $1 \text{ radian} = \frac{180^0}{\pi} = 57°16'$ (approx)

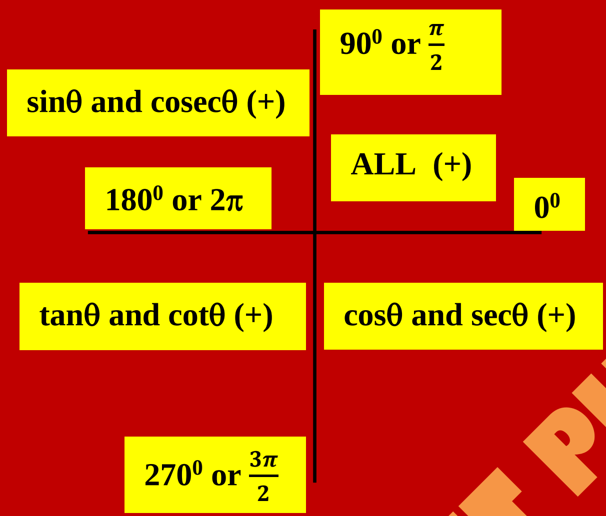
3. $1° = \frac{\pi}{180}$ radian = 0.01746 radians

4. $\theta = \frac{\text{arc}}{\text{radian}} = \frac{l}{r}$ or $l = r\theta$.

5. Radian measure = $\frac{\pi}{180^0} \times$ Degree measure

6. Degree measure = $\frac{180^0}{\pi} \times$ Radian measure

Trigonometric functions



First quadrant ($0^0 - 90^0$)

1. $\sin(90-\theta) = \cos\theta$; $\cos(90-\theta) = \sin\theta$

2. $\tan(90-\theta) = \cot\theta$; $\cot(90-\theta) = \tan\theta$

3. $\sec(90-\theta) = \operatorname{cosec}\theta$; $\operatorname{cosec}(90-\theta) = \sec\theta$

Second quadrant ($90^0 - 180^0$)

4. $\sin(90+\theta) = \cos\theta$; $\cos(90+\theta) = -\sin\theta$

5. $\operatorname{cosec}(90+\theta) = -\sec\theta$; $\sec(90+\theta) = -\operatorname{cosec}\theta$

6. $\tan(90+\theta) = -\cot\theta$; $\cot(90+\theta) = -\tan\theta$

7. $\sin(180-\theta) = \sin\theta$; $\cos(180-\theta) = -\cos\theta$

8. $\operatorname{cosec}(180-\theta) = \operatorname{cosec}\theta$; $\sec(180-\theta) = -\sec\theta$

9. $\tan(180-\theta) = -\tan\theta$; $\cot(180-\theta) = -\cot\theta$

Third quadrant ($180^0 - 270^0$)

10. $\sin(180+\theta) = -\sin\theta$; $\cos(180+\theta) = -\cos\theta$

11. $\operatorname{cosec}(180+\theta) = -\operatorname{cosec}\theta$; $\sec(180+\theta) = -\sec\theta$

12. $\tan(180+\theta) = \tan\theta$; $\cot(180+\theta) = \cot\theta$

13. $\sin(270-\theta) = -\cos\theta$; $\cos(270-\theta) = -\sin\theta$

14. $\operatorname{cosec}(270-\theta) = -\sec\theta$; $\sec(270-\theta) = -\operatorname{cosec}\theta$

15. $\tan(270-\theta) = \cot\theta$; $\cot(270-\theta) = \tan\theta$

Fourth quadrant ($270^0 - 360^0$)

16. $\sin(270+\theta) = -\cos\theta$; $\cos(270+\theta) = \sin\theta$

17. $\operatorname{cosec}(270+\theta) = \sec\theta$; $\sec(270+\theta) = \operatorname{cosec}\theta$

18. $\tan(270+\theta) = -\cot\theta$; $\cot(270+\theta) = -\tan\theta$

19. $\sin(360-\theta) = -\sin\theta$; $\cos(360-\theta) = \cos\theta$

20. $\operatorname{cosec}(360-\theta) = -\operatorname{cosec}\theta$; $\sec(360-\theta) = \sec\theta$

21. $\tan(360-\theta) = -\tan\theta$; $\cot(360-\theta) = -\cot\theta$

Trigonometric formulae for angles

1. $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$, $\cos\theta = \frac{1}{\sec\theta}$, $\tan\theta = \frac{1}{\cot\theta}$

2. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

3. $\sin^2\theta + \cos^2\theta = 1$

4. $\sec^2\theta = 1 + \tan^2\theta$

5. $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

6. $\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

7. $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

8. $\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

9. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

10. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$;

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11. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta;$

12. $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

13. $\tan(\theta+\phi) = \dfrac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$

14. $\tan(\theta-\phi) = \dfrac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$

15. $\cot(\theta+\phi) = \dfrac{\cot\theta\cot\phi - 1}{\cot\theta + \cot\phi}$

16. $\cot(\theta-\phi) = \dfrac{\cot\theta\cot\phi + 1}{\cot\theta - \cot\phi}$

17. $\sin(-\theta) = -\sin\theta;$

18. $\cos(-\theta) = \cos\theta;$

19. $\tan(-\theta) = -\tan\theta$

20. $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

21. $\sin(\theta-\phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$

22. $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

23. $\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

24. $2\sin\theta\cos\phi = \sin(\theta+\phi) + \sin(\theta-\phi)$

25. $2\cos\theta\sin\phi = \sin(\theta+\phi) - \sin(\theta-\phi)$

26. $2\cos\theta\cos\phi = \cos(\theta+\phi) + \cos(\theta-\phi)$

27. $2\sin\theta\sin\phi = \cos(\theta-\phi) - \cos(\theta+\phi)$

28. $\sin\theta + \sin\phi = 2\sin\left(\dfrac{\theta+\phi}{2}\right)\cos\left(\dfrac{\theta-\phi}{2}\right)$

29. $\sin\theta - \sin\phi = 2\cos\left(\dfrac{\theta+\phi}{2}\right)\sin\left(\dfrac{\theta-\phi}{2}\right)$

30. $\cos\theta + \cos\phi = 2\cos\left(\dfrac{\theta+\phi}{2}\right)\cos\left(\dfrac{\theta-\phi}{2}\right)$

31. $\cos\theta - \cos\phi = -2\sin\left(\dfrac{\theta+\phi}{2}\right)\sin\left(\dfrac{\theta-\phi}{2}\right)$

32. $\sin(\theta+\phi)\sin(\theta-\phi) = \sin^2\theta - \sin^2\phi$

33. $\sin(\theta+\phi)\sin(\theta-\phi) = \cos^2\phi - \cos^2\theta$

34. $\cos(\theta+\phi)\cos(\theta-\phi) = \cos^2\theta - \sin^2\phi$

35. $\cos(\theta+\phi)\cos(\theta-\phi) = \cos^2\phi - \sin^2\theta$

Trigonometric equations

The solutions of a trigonometric equation for which $0 \le x \le 2\pi$ are called principal solutions. The expression involving integer 'n' which gives all solutions of a trigonometric equation is called the general solution.

1. $\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y,$
   Where $n \in Z$ and $\forall\ x, y \in R$
2. $\cos x = \cos y, \Rightarrow x = 2n\pi \pm y,$
   Where $n \in Z$ and $\forall\ x, y \in R$
3. if x and y are not odd multiple of $\pi/2$, then $\tan x = \tan y \Rightarrow x = n\pi + y,$
   Where $n \in Z$ and $\forall\ x, y \in R$

Domain and Range of trigonometric functions

sin x	$R \to [-1, 1]$
cos x	$R \to [-1, 1]$
tan x	$R - \{x : x = (2n+1)\frac{\pi}{2}, n \in Z\} \to R$
cot x	$R - \{x : x = n\pi, n \in Z\} \to R$
sec x	$R - \{x : x = (2n+1)\frac{\pi}{2}, n \in Z\} \to R - (-1, 1)$
cosec x	$R - \{x : x = n\pi, n \in Z\} \to R - (-1, 1)$

Law of sines or sine formula:

In Δ ABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ = constant

Where a, b and c are the sides of the triangle.

Law of cosines or cosine formula:

In Δ ABC, $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$, $\cos B = \dfrac{a^2+c^2-b^2}{2ac}$

$\cos C = \dfrac{b^2+a^2-c^2}{2ab}$

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