Differential calculus at a glance

IMPORTANT LIMITS

1. $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$
2. $\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a$
3. $\lim_{x\to 0}\frac{e^x-1}{x}=\log_e e=1$
4. $\lim_{x\to 0}\frac{\sin x}{x}=1$
5. $\lim_{x\to 0}\frac{\tan x}{x}=1$
6. $\lim_{x\to 0}\frac{\ln(1+x)}{x}=1$
7. $\lim_{x\to 0}\frac{\sin^{-1}x}{x}=1$
8. $\lim_{x\to 0}\frac{\tan^{-1}x}{x}=1$

Continuity at a point

f(x) is continuous at a point x = c if

$\lim_{x\to c^+}f(x)=\lim_{x\to c^-}f(x)=f(c)$

If $\lim_{x\to c^+}f(x)=\lim_{x\to c^-}f(x)$,

it means limit at x = c exists

Differentiability at a point

f(x) is said to differentiable at x = c,

iff $\lim_{x\to c}f(x)=\frac{f(x)-f(c)}{x-c}$ exists finitely

i.e. Lf' (c) = R f' (c)

$$Lf'(c)=\lim_{x\to c^-}=\frac{f(x)-f(c)}{x-c}=\lim_{h\to 0}=\frac{f(c-h)-f(c)}{-h}$$

$$Rf'(c)=\lim_{x\to c^+}=\frac{f(x)-f(c)}{x-c}=\lim_{h\to 0}=\frac{f(c+h)-f(c)}{h}$$

Important Formulae of Differentiation

- $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$
- $\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$
- $\frac{d}{dx}[f(x).g(x)]=\left\{\frac{d}{dx}f(x)\right\}.g(x)+\left\{\frac{d}{dx}g(x)\right\}.f(x)$
- $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{\left\{\frac{d}{dx}f(x)\right\}.g(x)-f(x).\left\{\frac{d}{dx}g(x)\right\}}{(g(x))^2}$

1. $\frac{d}{dx}(x^n)=nx^{n-1}$
2. $\frac{d}{dx}(a^x)=(\ln a)a^x$
3. $\frac{d}{dx}(e^x)=e^x$
4. $\frac{d}{dx}(\ln x)=\frac{1}{x}$
5. $\frac{d}{dx}(\sin x)=\cos x$
6. $\frac{d}{dx}(\cos x)=-\sin x$
7. $\frac{d}{dx}(\tan x)=\sec^2 x$
8. $\frac{d}{dx}(\cot x)=-\operatorname{cosec}^2 x$
9. $\frac{d}{dx}(\sec x)=\sec x.\tan x$
10. $\frac{d}{dx}(\operatorname{cosec} x)=-\operatorname{cosec} x.\cot x$
11. $\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$
12. $\frac{d}{dx}(\cos^{-1}x)=\frac{-1}{\sqrt{1-x^2}}$
13. $\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$

14. $\frac{d}{dx}\left(\cot^{-1}x\right)=\frac{-1}{1+x^2}$

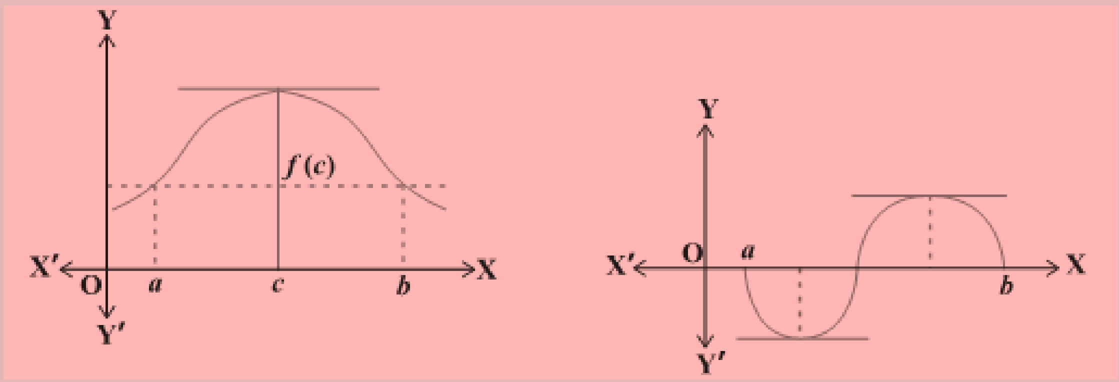
15. $\frac{d}{dx}\left(\sec^{-1}x\right)=\frac{1}{|x|\sqrt{x^2-1}}$

16. $\frac{d}{dx}\left(\operatorname{cosec}^{-1}x\right)=\frac{-1}{|x|\sqrt{x^2-1}}$

Rolle's Theorem:

Let f : [a, b] be continuous on [a, b] and differentiable on (a, b), such that f(a) = f(b), where a and b are some real numbers. Then there exists some c in (a, b) such that $f'(c) = 0$.

Following graphs of a few typical differentiable functions satisfying the hypothesis of Rolle 's Theorem are given.



Mean Value Theorem (MVT): Let f : [a, b] → R be a continuous function on [a, b] and differentiable on (a, b). Then there exists some c in (a, b) such that

$f'(c)=\frac{f(b)-f(a)}{b-a}$ MVT is an extension of Rolle's Theorem.